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CALCULATION OF NaI(Tl) DETECTOR FULL-ENERGY PEAK EFFICIENCY USING THE EFFICIENCY TRANSFER METHOD FOR SMALL RADIOACTIVE CYLINDRICAL SOURCES

by

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A direct analytical mathematical method is introduced to calculate the efficiency of gamma ray cylindrical detectors. The efficiency expression is deduced through a straightforward mathematical approach. The presented method is based on the accurate analytical calculation of the average path length covered by the photon within the detector's active volume, effective solid angle, and the efficiency transfer method in an integral form, so as to obtain a simple formula for the detection efficiency. In addition, the self-attenuation coefficient of the source matrix, the attenuation factors of the source container (with a radius smaller than the detector radius) and the detector housing materials are also treated by calculating the average path length within these materials. ^{152}Eu aqueous radioactive sources covering the energy range from 121 keV to 1408 keV were used. Remarkable agreement between the measured and the calculated efficiencies was achieved, with discrepancies less than 6 %.

Key words: NaI(Tl) scintillation detector, cylindrical source, full-energy peak efficiency, self-attenuation factor

Introduction

Absolute efficiency is greatly important in gamma ray spectroscopy, where it is needed to determine the activity of gamma-emitting radionuclides in a wide energy range. Cylindrical NaI(Tl) detectors are commonly used to identify and measure activities of low-level radioactive sources, because of their high detection efficiency and the fact that they operate at room temperature [1].

One of the main problems in gamma ray spectrometry is the accurate determination of the full-energy peak efficiency curve for a given sample matrix and measurement geometry. This problem is especially important in volumetric samples, where the self-attenuation effects are significant for a wide range of energies [2]. The most accurate method for calculating detector efficiency is the experimental method, which is limited by several factors, when the energy interval is broad. It requires a large number of radioactive standards, implying a high financial cost, along with counting time and a lot of effort in preparing the sources [3]. In order to overcome the said obstacles, several non-experimental methods [4-12] have been proposed and applied, depending on photon energy, source-detector geometry and volume. Also, Selim and Abbas overcame these hurdles by deriving a direct mathematical method for obtaining the efficiencies of source-detector systems with different geometries such as point source [13, 14], disk source [15, 16], cylindrical source [17], Marinelli beaker [18], parallelepiped [19], and well-type detectors [20]. Moreover, they introduced a new theoretical approach [21-25] based on the Direct Statistical Method to determine detector efficiency for an isotropic radiating point source at any arbitrary position from a cylindrical detector, as well as the extension of this approach using volumetric sources.

Badawi and co-workers deduced an analytical approach for calculating the full-energy peak efficiency of coaxial semiconductor detectors, including the source self-attenuation correction, attenuation by the source container and detector housing materials [26-30]. Hamzawy also derived an expression to calculate the detector total efficiency using off- axis point sources and coaxial circular disc sources with a cylindrical NaI(Tl) detector. These expressions are shorter than those in previous studies and easier to calculate, being of an elliptical integrations type and, thus, shortening, the program length, running time and increasing the accuracy of the calculations [31].

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One of the most useful techniques to calculate detector efficiency is the efficiency transfer method, in which the computation of detector efficiency for various geometrical conditions is derived from the known efficiency for the reference source-to-detector geometry [32-37]. In the present work, the authors introduce a new method, which depends on three important factors. The first one is the efficiency transfer method. The second one is the accurate analytical calculation of the average path length covered by the photon in each of the following: detector active volume, source matrix, source container, dead layer and the end cap of the detector. The third uses the technique introduced by Hamzawy [31].

MATHEMATICAL TREATMENT

The efficiency transfer principle, as estimated in [38], was applied to establish the efficiency calibration curves of the cylindrical detectors based on the following equation.

$$\varepsilon_{\text{target}} = \frac{\Omega_{\text{target}}}{\Omega_{\text{ref}}} \varepsilon_{\text{ref}} \qquad (1)$$

where $\varepsilon_{\text{target}}$ and ε_{ref} are the full-energy peak efficiency (FEPE) of the target (small radioactive cylindrical source) and the reference geometry (an isotropic radiating point source placed on the crystal axes), respectively. Ω_{target} and Ω_{ref} are the effective solid angles subtended by the detector surface with the target and the reference point source, respectively. In order to use the efficiency transfer principle, the experimental reference efficiency ε_{ref} was essentially measured [39, 40].

The effective solid angle for the isotropic coaxial radioactive point source $\Omega_{\text{eff(point)}}$ can be calculated by assuming that, there are two main cases to be considered, as indicated in fig. 1. The striking photon may enter the upper face of the cylindrical detector (with radius R and length L), and may emerge from its opposite base or from its side with different path lengths (d_1 and d_2, respectively) and can be given by

$$d_1 = \frac{L}{\cos\theta} \qquad (2)$$

$$d_2 = \frac{\rho\cos\varphi + \sqrt{R^2 - \rho^2\sin^2\varphi}}{\sin\theta} - \frac{h}{\cos\theta} \qquad (3)$$

Consider the detector has a reflective layer covering its upper surface with thickness t_{RL}, that each photon entering the detector must pass through the upper surface of the reflective layer and, consequently, the photon path length t_{dl} through the upper surface reflective layer can be given by

$$t_{\text{dl}} = \frac{t_{\text{RL}}}{\cos\theta} \qquad (4)$$

Also, if t_{ecf} is the thickness of the detector end cap material, the photon path length t_{ecl} through the end cap material can be given by
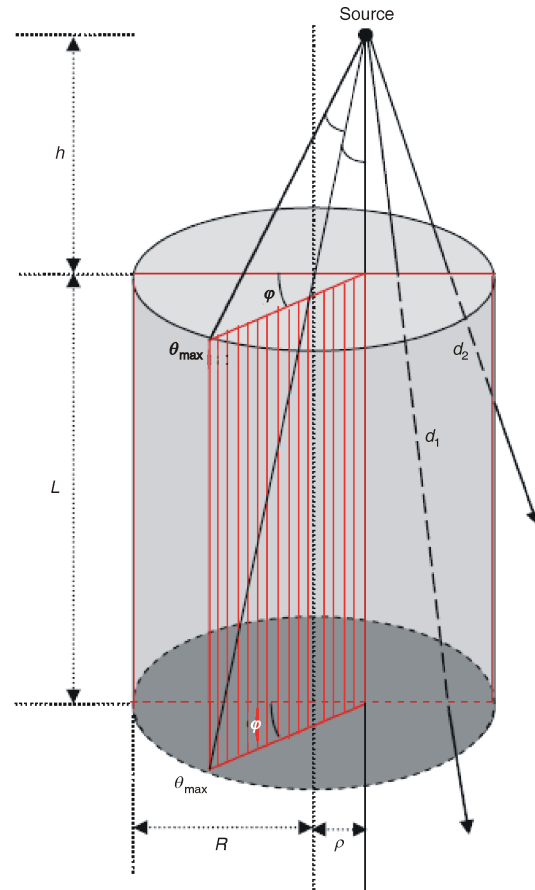

Figure 1. A diagram of a cylindrical-type detector with a non-axial point source ($\rho < R$)

$$t_{\text{ecl}} = \frac{t_{\text{ecf}}}{\cos\theta} \qquad (5)$$

The polar angles can take the values as follow

$$\theta_{\max} = \tan^{-1}\frac{\rho\cos\varphi + \sqrt{R^2 - \rho^2\sin^2\varphi}}{h}$$

$$\theta'_{\max} = \tan^{-1}\frac{\rho\cos\varphi + \sqrt{R^2 - \rho^2\sin^2\varphi}}{h+L} \qquad (6)$$

where $\theta_{\max}$ and $\theta'_{\max}$ are the maximum polar angles for the photon to enter from the detector face and exit from the detector base, respectively.

For the coaxial radioactive point source, the lateral distance ρ is equal to zero and, according to the present symmetry, the maximum azimuthal angles φ are equal to 2π. Therefore, the effective solid angle for an axial radioactive point source can be expressed by

$$\Omega_{\text{eff(point)}} = f_{\text{att}} f \Omega_{\text{pure(point)}} \qquad (7)$$

where $\Omega_{\text{pure(point)}}$ is the solid angle subtended by the radioactive point source and the active surface of the detector and can be defined by

$$\Omega_{\text{pure(point)}} = \int_{\varphi}\int_{\theta}\sin\theta\,d\theta\,d\varphi = \int_0^{2\pi}\int_0^{\theta_{\max}}\sin\theta\,d\theta\,d\varphi \qquad (8)$$

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and

$$f = 1 - e^{-\mu \bar{d}} \tag{9}$$

where μ is the detector total attenuation coefficient without coherent scattering and $\bar{d}$ – the average path length traveled by a photon through the detector medium and can be given by

$$\bar{d} = \frac{\int_\Omega d(\theta,\varphi)\mathrm{d}\Omega}{\int_\Omega \mathrm{d}\Omega} = \frac{\int_0^{2\pi}\int_0^{\theta_{max}} d_1 \sin\theta\mathrm{d}\theta + \int_0^{\theta_{max}} d_2 \sin\theta\mathrm{d}\theta \ \mathrm{d}\theta}{\int_0^{2\pi}\int_0^{\theta_{max}} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi} \tag{10}$$

The factor f_{att} determines the photon attenuation by all absorbers between the source and the detector and can be given by

$$f_{att} = f_{Rlay} \ f_{Ecap} \tag{11}$$

where f_{Rlay} is the attenuation by the reflective layer, while f_{Ecap} is the attenuation of the end cap material and both can be given by

$$f_{Rlay} = e^{-\mu_{Rlay}\bar{\delta}_{Rlay}}$$
$$f_{Ecap} = e^{-\mu_{Ecap}\bar{\delta}_{Escap}} \tag{12}$$

where μ_{Rlay} and μ_{Ecap} are the attenuation coefficients of the reflective layer and end cap, respectively, while $\bar{\delta}_{Rlay}$ and $\bar{\delta}_{Ecap}$ are the average path length traveled by a photon through the reflective layer and the end cap, respectively, which can be given by

$$\bar{\delta}_{Dlay} = \frac{\int_\varphi\int_\theta t_d(\theta,\varphi)\sin\theta\mathrm{d}\theta\mathrm{d}\varphi}{\int_\varphi\int_\theta \sin\theta\mathrm{d}\theta\mathrm{d}\varphi} = \frac{\int_0^{2\pi}\int_0^{\theta_{max}} t_{d1} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi}{\int_0^{2\pi}\int_0^{\theta_{max}} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi}$$

$$\bar{\delta}_{Ecap} = \frac{\int_\varphi\int_\theta t_{ec}(\theta,\varphi)\sin\theta\mathrm{d}\theta\mathrm{d}\varphi}{\int_\varphi\int_\theta \sin\theta\mathrm{d}\theta\mathrm{d}\varphi} = \frac{\int_0^{2\pi}\int_0^{\theta_{max}} t_{ecl} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi}{\int_0^{2\pi}\int_0^{\theta_{max}} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi} \tag{13}$$

The radioactive cylindrical source (with radius S and height H) can be considered as a volumetric source, as shown in fig. 2, where it consists of a group of uniformly distributed radioactive point sources, each with its own effective solid angle, $\Omega_{eff(point)}$. Thus, the total effective solid angle $\Omega_{eff(cyl)}$ of the cylindrical detector when using a radioactive cylindrical source can be given by

$$\Omega_{Ef(Cyl)} = f_{att} S_{self} S_{sc} f\Omega_{Pure(Cyl)} \tag{14}$$

where

$$\Omega_{Pure(Cyl)} = \frac{\int_v \Omega_{Pure(Point)}\mathrm{d}V}{V} \tag{15}$$
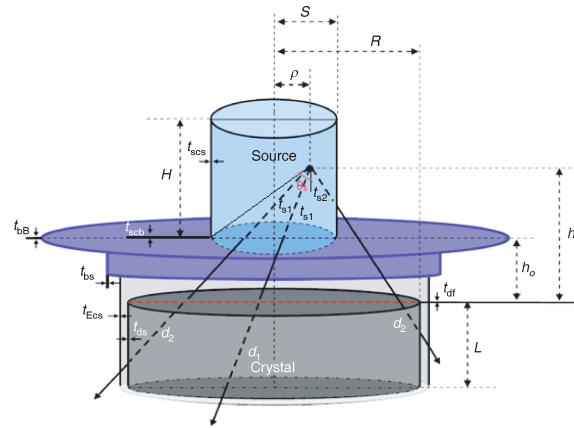

Figure 2. Possible cases of photon path lengths through a source-to-detector system

where V is the volume of the radioactive source. Any element of volume $\mathrm{d}V = \rho \ \mathrm{d}\rho \ \mathrm{d}\alpha \ \mathrm{d}h$, when displaced at a lateral distance ρ from the detector axis and emitting a photon at an angle α to the detector major axis, can be expressed in cylindrical co-ordinates and eq. (15) can be rewritten as

$$\Omega_{Pure(Cyl)} = \frac{\int_H\int_{h_o}\int_{2\pi S}\int_h\int_\alpha\int_\rho \Omega_{Pure(Point)}\rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h}{\pi S^2 H} = $$
$$= \frac{\int_{h_0}\int_0\int_0 \Omega_{Pure(Point)}\rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h}{\pi S^2 H} \tag{16}$$

The factors f and f_{att} have been defined in eqs. (9) and (11), but in this case, the average path length, $\bar{d}$, traveled by the photon through the detector and the solid angle will assume new forms due to the geometry of fig. 2. The average path length can be expressed by

$$\bar{d} = \frac{\int_h\int_\alpha\int_\rho\int_\Omega d(\theta,\varphi)\mathrm{d}\Omega\rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h}{\int_h\int_\alpha\int_\rho\int_\Omega \mathrm{d}\Omega\rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h} =$$

$$= \frac{\int_{h_o}^H\int_0^{h_o}\int_0^{2\pi S}\int_0^{2\pi}\int_0^{\theta_{max}}\left[d_1\sin\theta\mathrm{d}\theta + \int^{\theta_{max}}_{\theta_{max}} d_2\sin\theta\mathrm{d}\theta\right]\mathrm{d}\varphi \ \rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h}{\int_{h_o}^H\int_0^{h_o}\int_0^{2\pi S}\int_0^{2\pi\theta_{max}} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi \ \rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h} \tag{17}$$

The new formulas of the average path length traveled by the photon through the detector reflective layer and the detector end cap material are given by eqs. (18) and (19), respectively.

$$\bar{\delta}_{Dlay} = \frac{\int_{h_o}^H\int_0^{h_o}\int_0^{2\pi S}\int_0^{2\pi}\int_0^{\theta_{max}} t_{d1} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi \ \rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h}{\int_{h_o}^H\int_0^{h_o}\int_0^{2\pi S}\int_0^{2\pi}\int_0^{\theta_{max}} \sin\theta\mathrm{d}\theta\mathrm{d}\varphi \ \rho\mathrm{d}\rho\mathrm{d}\alpha\mathrm{d}h} \tag{18}$$

$$\bar{\delta}_{Ecap} = \frac{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S\int_0^{2\pi}\int_0^{\theta_{max}} t_{ec1}\sin\theta\,d\theta\,d\varphi\,\rho\,d\rho\,d\alpha\,dh}{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S\int_0^{2\pi}\int_0^{\theta_{max}}\sin\theta\,d\theta\,d\varphi\,\rho\,d\rho\,d\alpha\,dh} \quad (19)$$

In the case of a radioactive cylindrical source with a radius smaller than that of the detector and h_o, being the distance between the source and the detector's upper surface, there are two possible paths, t_{s1} and t_{s2}, for the photon to leave the source which can be given as:

(1) To exit from the radioactive source base

$$t_{s1} = \frac{h - h_o}{\cos\theta} \quad (20)$$

(2) To exit from the radioactive source side

$$t_{s2} = \frac{-\rho\cos\varphi + \sqrt{S^2 - \rho^2\sin^2\varphi}}{\sin\theta} \quad (21)$$

The maximum polar angle θ_s to let the photon exit from the radioactive source base can be given by

$$\theta_s = \tan^{-1}\frac{-\rho\cos\varphi + \sqrt{S^2 - \rho^2\sin^2\varphi}}{h - h_o} \quad (22)$$

The self-attenuation factor S_{self} of the radioactive source matrix is given by

$$S_{self} = e^{-\mu_s \bar{t}_s} \quad (23)$$

where μ_s is the attenuation coefficient of the source matrix and $\bar{t}_s$ – the average path length traveled by the photon inside the source and can be given by

$$\bar{t}_s = \frac{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S g_{sl}\,\rho\,d\rho\,d\alpha\,dh}{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S\int_0^{2\pi}\int_0^{\theta_{max}}\sin\theta\,d\theta\,d\varphi\,\rho\,d\rho\,d\alpha\,dh} \quad (24)$$

There are two possible values of g_{sl} according to the values of the polar angle θ which can be given by

$$g_{sl} = \int_0^{2\pi}\left[\int_0^{\theta_s} t_{s1}\sin\theta\,d\theta + \int_{\theta_s}^{\theta_{max}} t_{s2}\sin\theta\,d\theta\right]d\varphi \quad (25)$$

for case in which $(\theta_s < \theta_{max})$, and

$$g_{sl} = \int_0^{2\pi}\int_0^{\theta_{max}} t_{s1}\sin\theta\,d\theta\,d\varphi \quad (26)$$

for case in which $(\theta_s \geq \theta_{max})$.

If t_{scb} is the source container bottom thickness and t_{scs} is the source container wall thickness, there are two possible path lengths, t_{sc1} and t_{sc2}, for the photon to exit from the source container:

(1) To exit from the radioactive source base

$$t_{sc1} = \frac{t_{scb}}{\cos\theta} \quad (27)$$

(2) To exit from the radioactive source side

$$t_{sc2} = \frac{-\rho\cos\varphi + \sqrt{(S + t_{scs})^2 - \rho^2\sin^2\varphi}}{\sin\theta} -$$
$$-\frac{-\rho\cos\varphi + \sqrt{S^2 - \rho^2\sin^2\varphi}}{\sin\theta} =$$
$$= \frac{t_{scs}\left(1 - \frac{\rho^2}{2S^2}\sin^2\varphi\right)}{\sin\theta} \quad (28)$$

The maximum polar angle θ_{sc} to let the photon exit from the radioactive source container base can be given by

$$\theta_{sc} = \tan^{-1}\frac{-\rho\cos\varphi + \sqrt{(S + t_{scs})^2 - \rho^2\sin^2\varphi}}{h - h_o + t_{scb}} \quad (29)$$

The attenuation factor S_{sc} of the radioactive source container material can be given by

$$S_{sc} = e^{-\mu_c \bar{t}_{sc}} \quad (30)$$

where μ_c is the attenuation coefficient of the radioactive source container material and $\bar{t}_{sc}$ is the average path length traveled by a photon inside the radioactive source container, which can be given by

$$\bar{t}_{sc} = \frac{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S g_{scl}\,\rho\,d\rho\,d\alpha\,dh}{\int_{h_o}^{H+h_o}\int_0^{2\pi}\int_0^S\int_0^{2\pi}\int_0^{\theta_{max}}\sin\theta\,d\theta\,d\varphi\,\rho\,d\rho\,d\alpha\,dh} \quad (31)$$

There are two cases for the values of g_{scl} according to the values of the polar angle θ and they can be given by

– The case in which $(\theta_{sc} < \theta_{max})$

$$g_{scl} = \int_0^{2\pi}\left[\int_0^{\theta_{sc}} t_{scl}\sin\theta\,d\theta + \int_{\theta_{sc}}^{\theta_{max}} t_{sc2}\sin\theta\,d\theta\right]d\varphi \quad (32)$$

The case in which $(\theta_{sc} \geq \theta_{max})$

$$g_{scl} = \int_0^{2\pi}\int_0^{\theta_{max}} t_{scl}\sin\theta\,d\theta\,d\varphi \quad (33)$$

By using the efficiency transfer principle, the full-energy peak efficiency of the cylindrical detector when using a coaxial radioactive cylindrical source can be calculated based on the reference measured full-energy peak efficiency of the cylindrical detector, with respect to an axial radioactive point source and can be given by the following formula

$$\varepsilon_{(Cyl)} = \frac{\Omega_{Eff(Cyl)}}{\Omega_{Eff(Point)}}\varepsilon_{(Point)} \quad (34)$$

where $\varepsilon_{(Cyl)}$ and $\varepsilon_{(Point)}$ are the full-energy peak efficiency for the cylindrical detector for using a radioactive cylindrical source and isotropic coaxial radioac-

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tive point source as the a reference geometry, respectively. While $\Omega_{\text{Eff(Cyl)}}$ and $\Omega_{\text{Eff(Point)}}$ are the effective solid angles subtended by the detector surface with the radioactive cylindrical source and the reference geometry, respectively.

EXPERIMENTAL SET-UP

The experimental section of this work was performed in Prof. Dr. Y. S. Selim, Laboratory for Radiation Physics, Physics Department, Faculty of Science, Alexandria University, Alexandria, Egypt. Full-energy peak efficiency values were determined for two NaI(Tl) detectors with resolutions of 8.5 % and 7.5 % at the 662 keV peak of ^{137}Cs, labeled as Det.1 and Det.2, respectively. The manufacturer parameters and setting up values are shown in tab. 1. The FEPE has been measured by using two different types of radioactive sources. These radioactive point sources are ^{241}Am, ^{133}Ba, ^{152}Eu, ^{137}Cs, and ^{60}Co, purchased from the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig and Berlin Germany. The sources' activities and their uncertainties, half-lives, photon energies and photon emission probabilities per decay for all PTB sources are listed in tab. 2.

The calibration process was done by the PTB radioactive point sources, where the homemade Plexiglas holder was used to measure these sources at four different axial distances from the detector surface, starting from $P_4 = 20$ cm up to $P_{10} = 50$ cm, with 10 cm at each step.

Radioactive cylindrical sources in polypropylene (PP) plastic vials with a volume of 25 ml and 50 ml, filled with an aqueous solution containing ^{152}Eu radionuclide, emitting γ-rays in the energy range from 121 keV to 1408 keV, served as the second source.

Table 3 shows the radioactive source dimensions. Efficiency measurements are carried out by putting the sources over the plexiglas holder, which secures the top of the end cap of the detector, as shown in fig. 2. In order to minimize the dead time, the activity of the sources was prepared to be (5050 ± 50 Bq). The measurements are carried out to obtain statistically significant main peaks in the spectra, recorded and processed by winTMCA32 software made by ICx Technologies. The measured spectrum saved as Spectrum ORTEC files can be opened by ISO 9001 Genie 2000 data acquisition and analysis software made by Canberra, Australia [41].

The acquisition time is high enough to get at least 20,000 counts which make the statistical uncertainties less than 0.7%. The spectra are analyzed with the program using its automatic peak search and peak area calculations, along with changes in the peakfit, using the interactive peakfit interface when necessary to reduce the residuals and errors in peak area values. The peak areas, live- time, runtime, and the start time for each spectrum, were entered in the spreadsheets that are used to perform the calculations necessary to generate the efficiency curves.

RESULTS AND DISCUSSION

The experimental FEPE at energy E for a given set of measuring conditions can be computed by the equation

$$\varepsilon(E) = \frac{N(E)}{t A_S P(E)} C_i \qquad (35)$$

Table 1. Manufacturer parameters and set-up values for the detectors

Items	Det. 1	Det. 2
Manufacturer	Canberra	Canberra
Serial number	09L 654	09L 652
Detector model	802	802
Type	Cylindrical	Cylindrical
Mounting	Vertical	Vertical
Resolution (FWHM) at 662 keV	8.5 %	7.5 %
Cathode to anode voltage	+1100 V d.c.	+1100 V d.c.
Dynode to dynode	+80 V d.c.	+80 V d.c.
Cathode to dynode	+150 V d.c.	+150 V d.c.
Tube base	Model 2007	Model 2007
Shaping mode	Gaussian	Gaussian
Detector type	NaI(Tl)	NaI(Tl)
Weight [kg]	0.77	1.8
Crystal volume [cm^3]	103	347.64
Crystal diameter [mm]	50.8	76.2
Crystal length [mm]	50.8	76.2
Top cover thickness [mm]	Al (0.5)	Al (0.5)
Side cover thickness [mm]	Al (0.5)	Al (0.5)
Reflector – oxide [mm]	2.5	2.5
Outer diameter [mm]	57.2	80.9
Outer length [mm]	53.9	79.4

Table 2. Point source activities and their uncertainties, half livfes, photon energies and photon emission probabilities per decay

PTB-nuclide	Energy [keV]	Emission probability [%]	Half life [days]	Activity [kBq] at June 1, 2009 00:00 Hr	Uncertainty [kBq]
^{241}Am	59.52	35.9	157861.05	259.0	2.6
^{133}Ba	80.99	34.1	3847.91	275.3	2.8
^{152}Eu	121.78	28.4	4943.29	290.0	±4.0
	244.69	7.49			
	344.28	26.6			
	778.90	12.96			
	964.13	14.0			
	1408.01	20.87			
^{137}Cs	661.66	85.21	11004.98	385.0	4.0
^{60}Co	1173.23	99.9	1925.31	212.1	1.5
	1332.5	99.982			

Table 3. Parameters of the radioactive cylindrical sources

Items	Source description	
	V1	V2
Volume [ml]	25	50
Outer diameter [mm]	32.1	39.658
Height [mm]	36.21	45.4
Wall thickness [mm]	1.2	1.1
Manufacturer	Azlon	
Jar material	Polypropylene	

where $N(E)$ is the number of counts in the full-energy peak, t – the measuring time (in seconds), $P(E)$ – the photon emission probability at energy E, A_S – the radionuclide activity, and C_i are the correction factors due to dead-time and radionuclide decay. Intended for measurements of the radioactive volumetric and point sources, the dead-time has always been less than 2 %, when the sources with high activities were placed at a secure enough distance from the detector surface in order to reduce the dead-time to this percentage. Therefore, the corresponding factor was obtained by simply using ADC real time. The decay correction C_d for the calibrating source from the reference time to the runtime is given by

$$C_d \quad e^{\lambda \Delta t} \qquad (36)$$

where λ is the decay constant and Δt – the time elapsed between the reference date and the time of measurement. The uncertainty in the experimental full-energy peak efficiency σ_ε is given by

$$\sigma_\varepsilon \quad \varepsilon \sqrt{\left(\frac{\partial \varepsilon}{\partial A}\right)^2 \sigma_A^2 \quad \left(\frac{\partial \varepsilon}{\partial P}\right)^2 \sigma_P^2 \quad \left(\frac{\partial \varepsilon}{\partial N}\right)^2 \sigma_N^2} \qquad (37)$$

where σ_A, σ_P, and σ_N are the uncertainties associated with the quantities A_S, $P(E)$ and $N(E)$, respectively, assuming that the only correction made is due to source activity decay.

The full-energy peak efficiency FEPE curves were determined for point radioactive sources for four measurement geometries (positions P4, P6, P8 and P10). Each of these curves was used to obtain a FEPE curve for two cylindrical sources (V1 and V2), measured at the top of the detector, by the ET method. The transferring process, as described, was performed for two NaI(Tl) detectors (Det. 1 and Det. 2). Also, the FEPE curves for the cylindrical source geometry were experimentally determined, and the comparison of experimental values and efficiencies obtained by the (ET) method for each detector presented in figs. 3 and 4 considered to serve as examples and a graphic display of the said results. The discrepancies between the calculated and the measured efficiencies, given by eq. (38), where ε_{cal} and ε_{meas} are the calculated and experimentally measured efficiencies, respectively, are listed in tab. 4.
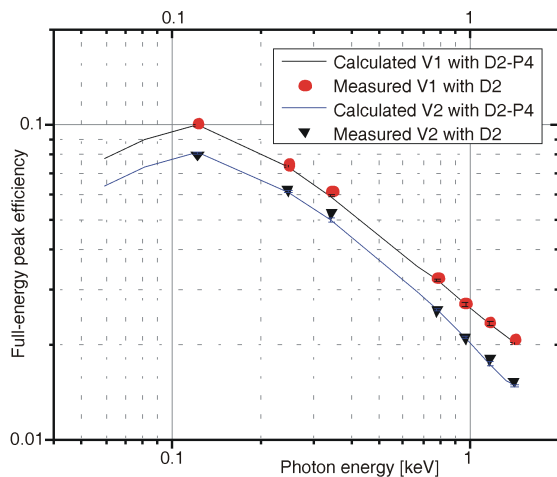
$$\Delta\% \quad \frac{\varepsilon_{cal} \quad \varepsilon_{meas}}{\varepsilon_{meas}} \quad 100 \qquad (38)$$
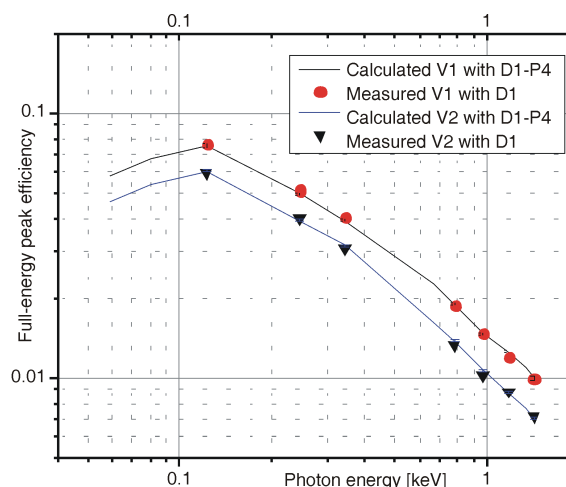
Figure 3. The calculated full-energy peak efficiency for detector Det. 1 using radioactive cylindrical sources V1 and V2, based on reference values at position P4, compared with the ones measured with their associated uncertainties as a function of photon energy

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Table 4. The discrepancy percentage (%) between the experimental and theoretical values of FEPE for detectors Det. 1 and Det. 2 using radioactive cylindrical sources V1 and V2 based reference values at positions P4, P6, P8, and P10

Energy [keV]	V1				V2			
(%) [Det. 1]								
	P4	P6	P8	P10	P4	P6	P8	P10
121.78	−1.84	1.05	1.59	−2.80	1.90	−0.79	1.07	0.98
244.69	−3.60	3.58	2.58	0.42	−2.25	0.64	−3.11	−0.86
344.28	−3.24	2.88	1.59	−1.22	2.11	3.86	−0.07	1.50
778.90	0.09	−1.19	−0.04	0.07	4.11	4.06	−0.30	0.46
964.13	−0.30	−1.45	−0.41	−0.57	4.89	4.97	0.54	1.03
1173.23	4.05	−2.60	4.06	3.87	−0.82	−0.85	−0.44	−3.08
1408.01	0.77	1.63	0.24	−2.45	−2.36	−2.99	−2.55	−0.32
(%) [Det. 2]								
Energy [keV]	V1				V2			
	P4	P6	P8	P10	P4	P6	P8	P10
121.78	−1.85	0.33	0.09	4.82	1.21	3.32	−3.84	−4.67
244.69	−1.76	1.93	3.11	2.72	−1.16	2.50	3.68	3.29
344.28	−4.24	−1.10	0.46	−0.85	−5.57	−2.39	−0.81	−2.14
778.90	−2.27	1.50	4.11	3.56	−4.15	3.39	−0.04	2.79
964.13	−1.40	−3.10	−0.52	0.05	−0.22	2.52	−1.31	−2.72
1173.23	1.67	−0.44	−2.20	0.77	−4.55	1.05	−1.55	−0.64
1408.01	−1.12	−2.96	0.68	−2.26	−1.17	0.25	−1.71	−1.85

Figure 4. The calculated full-energy peak efficiency for detector Det. 2 using radioactive cylindrical sources V1 and V2, based on reference values at position P4, compared with the ones measured and associated uncertainties as a function of photon

CONCLUSIONS

This paper presents a simple analytical method to evaluate the full-energy peak efficiency covering a wide energy range while, using isotropic axial radioactive point sources, and axial radioactive cylindrical sources. The present approach offers a great possibility to calibrate the detectors through the determination of the FEPE curve, even in cases when no standard source is available, this being the ultimate goal of our work. In general, the discrepancies between the calculated and the experimental values of all measurements were found to be less than 6 % across the entire energy range of interest.

AUTHORS' CONTRIBUTIONS

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ПРОРАЧУН ЕФИКАСНОСТИ NAI(Tl) ДЕТЕКТОРА
У ПИКУ ЕНЕРГЕТСКЕ РАСПОДЕЛЕ МЕТОДОМ ТРАНСФЕРА ЕФИКАСНОСТИ
ЗА МАЛЕ ЦИЛИНДРИЧНЕ РАДИОАКТИВНЕ ИЗВОРЕ

Приказана је директна аналитичко-математичка метода за прорачун ефикасности цилиндричних детектора гама зрачења и изведен је израз за ефикасност директним математичким поступком. Представљена метода заснива се на тачном аналитичком прорачуну средње дужине слободног пута фотона у активној запремини детектора, ефективном просторном углу и методи трансфера ефикасности у интегралном облику – са циљем да се добије једноставна формула за ефикасност детектора. Коефицијент само-слабљења матрице извора, фактори слабљења контејнера извора (са радијусом мањим од радијуса детектора) и материјала кућишта детектора, такође су разматрани израчунавањем средњег слободног пута у овим материјалима. Коришћени су течни радиоактивни извори ^{152}Eu који покривају распон енергија од 121 keV до 1408 keV. Постигнуто је изванредно слагање мерене и рачунате ефикасности, са разликама мањим од 6 %.

Кључне речи: NaI(Tl) сцинтилациони детектор, цилиндрични извор, ефикасност у пику енергије,
фактор само-слабљења